MPAe contributions to the SECCHI 3D reconstruction software package

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The MPAe will get engaged in three areas:

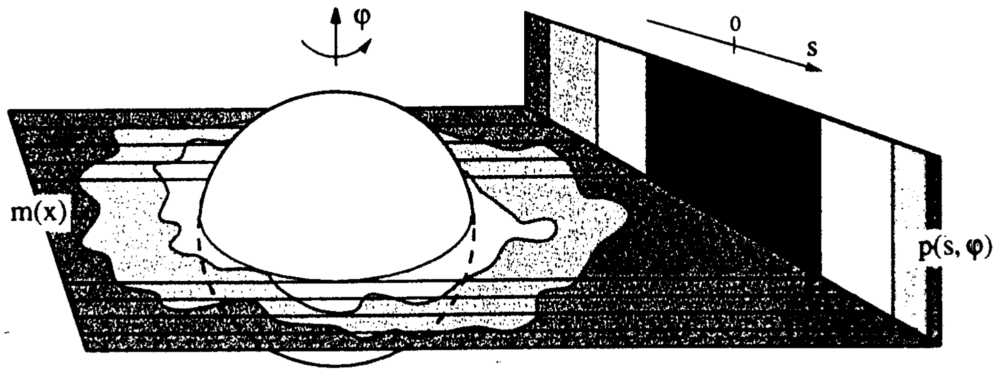
- Tomography of coronal data
- Stereoscopy of loop systems
- Force free magnetic field modelling

$\Longrightarrow$ Outline of present concepts

$\Longrightarrow$ Problems and parameters a user should be aware of

Coronal Tomography

- The inversion of the X-ray transform is an ill-posed problem. The condition number is especially large if data from inside the occulter is missing (Natterer, 1986).



The emission density $\boldsymbol{m}$ and image intensity $\boldsymbol{p}$ are related for $|s| > \mathrm{R}$ (exterior problem) by

$$\boldsymbol{p}(s,\varphi) = \int_{\boldsymbol{x}\cdot \boldsymbol{e}_{\varphi}^{\perp}=s} \boldsymbol{m}(\boldsymbol{x})\, d\boldsymbol{x} \ \stackrel{grid}{\longrightarrow}\ \boldsymbol{A}(s,\varphi;\boldsymbol{x}) \cdot \boldsymbol{m}(\boldsymbol{x})$$

where $\boldsymbol{e}_{\varphi}^{\perp}$ is perpendicular to the Sun's rotation axis and the line of sight.

$\Longrightarrow$ The inversion of this integral equation is extremely unstable and requires special numerical techniques, e.g., regularization (Tikhonov, 1963):

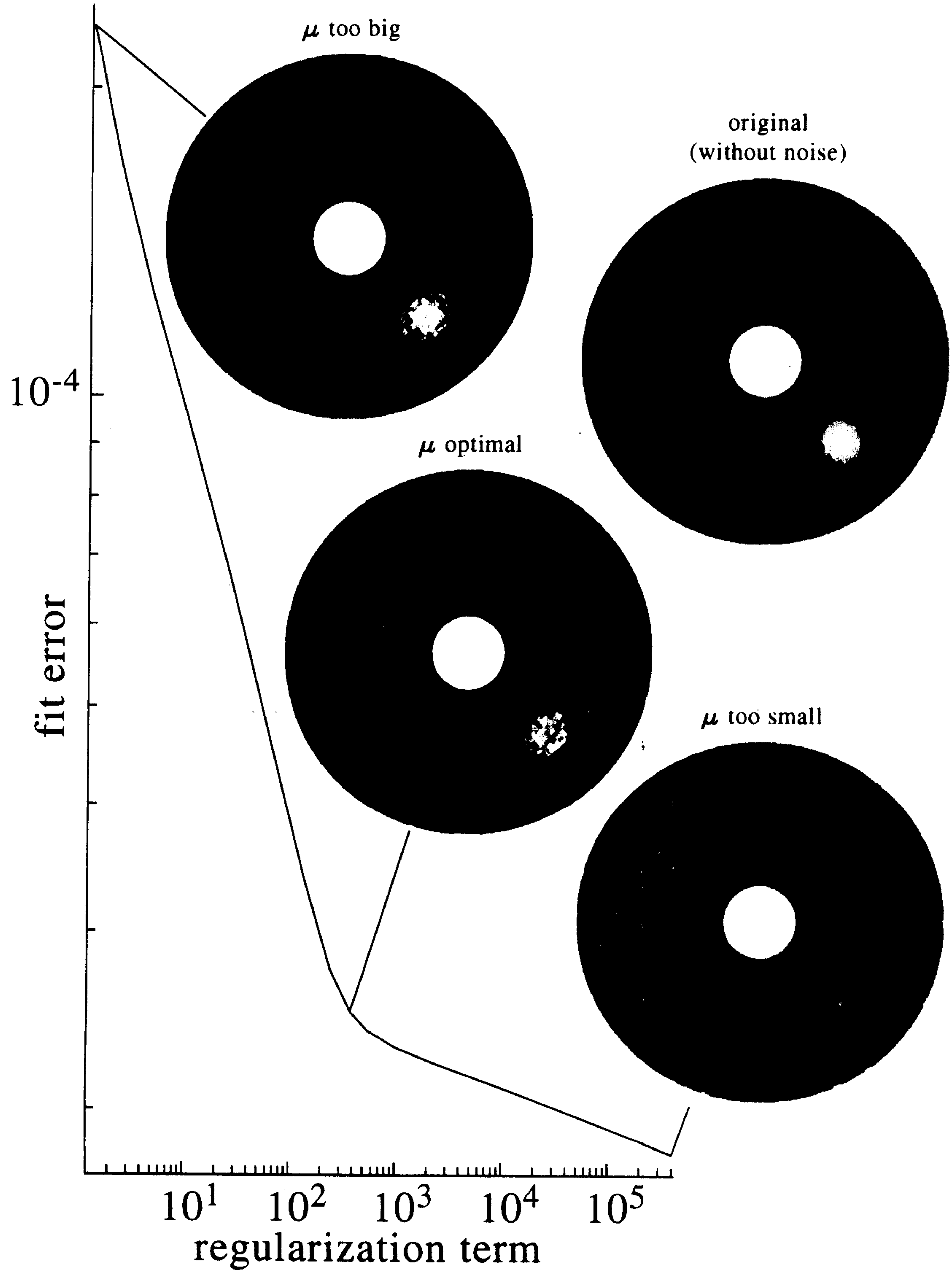
$$\|\boldsymbol{p}(s,\varphi) - \boldsymbol{A}(s,\varphi;\boldsymbol{x}) \cdot \boldsymbol{m}(\boldsymbol{x})\|^2$$

$$+\mu\ \|\text{integral constraint on } \boldsymbol{m}\|^2$$

$$\longrightarrow \text{minimum}$$

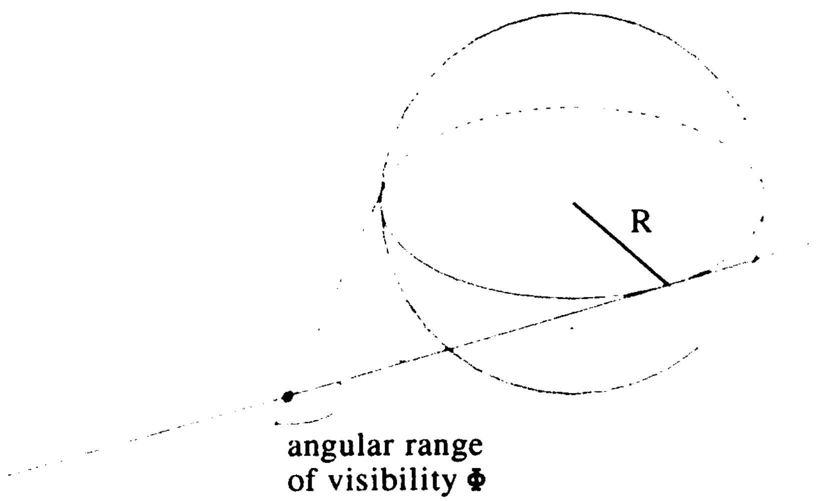
The regularization term may include structural information, e.g., magnetic field direction.

Optimal regularization parameter μ from the L-curve (Hansen, 1992)



Decrease of resolution due to occultation

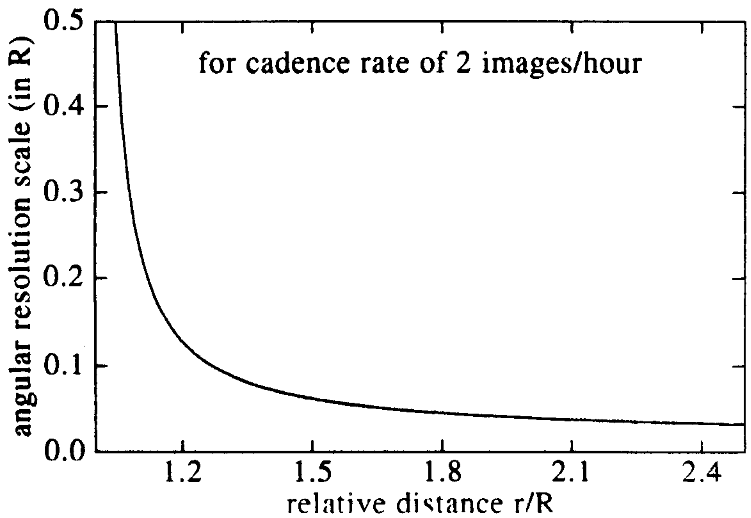
- Data missing from inside the occulter causes a severe decrease of angular resolution close to the occulter surface:



The maximum spatial resolution that can be achieved by tomographic reconstruction of occulted images (exterior problem; Zidowitz, 1997)

$$\Delta_r \simeq \text{pixel size} \cdot 2$$

$$\Delta_\phi \simeq \frac{4\pi R_{\max}}{\text{images per rotation}} \left(\frac{\pi}{\Phi}\right)^2, \quad \Phi = \pi - 2\,\mathrm{asin}\left(\frac{R}{r}\right)$$

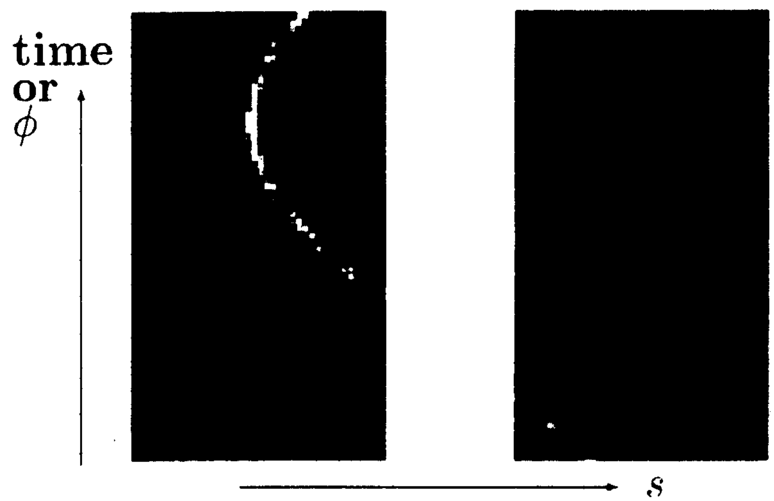


Inconsistency of data

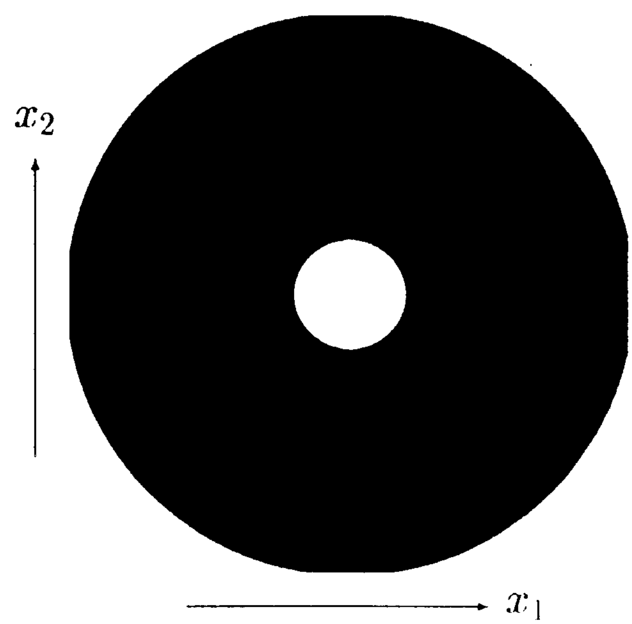
- Measurement errors and/or time variation of coronal structures may cause the observed data $\boldsymbol{p} \notin \text{range}(\boldsymbol{A})$

$\Rightarrow$ Preprocessing of data (δ-element as an example):

original data



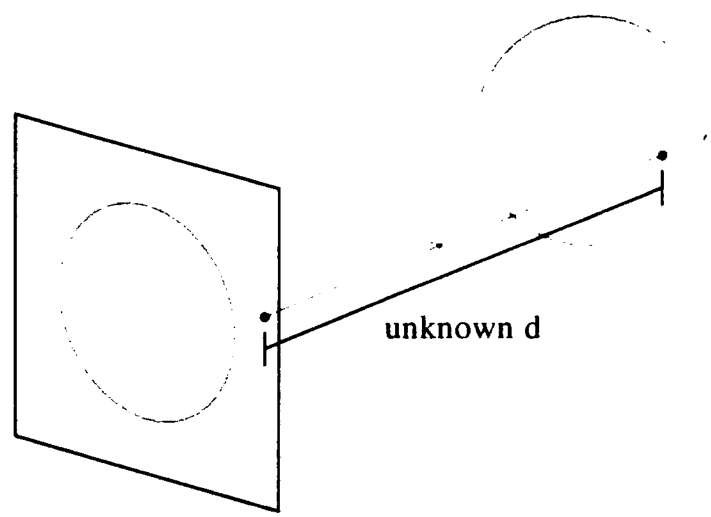
preprocessed data $= \boldsymbol{A}^T\boldsymbol{p}$



The preprocessed data is consistent.
In case of a time variation it represents essentially the average over the visible path of a volume element in $\boldsymbol{x}$-space

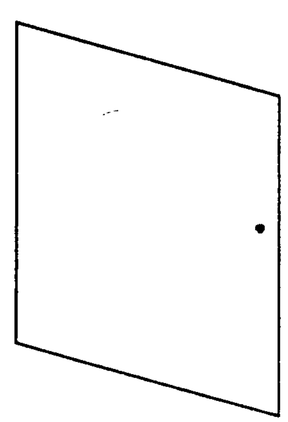
Stereoscopy vs tomography

- Fundamental assumption in stereoscopy:



Emission from a single spot along line of sight → a second view is sufficient to determine d.

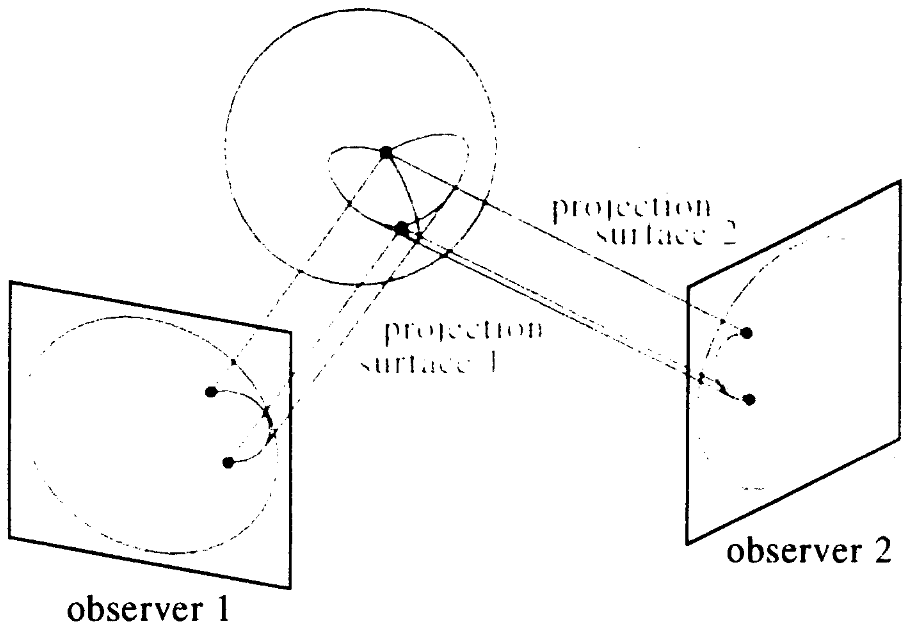
- Generalzation in tomography:



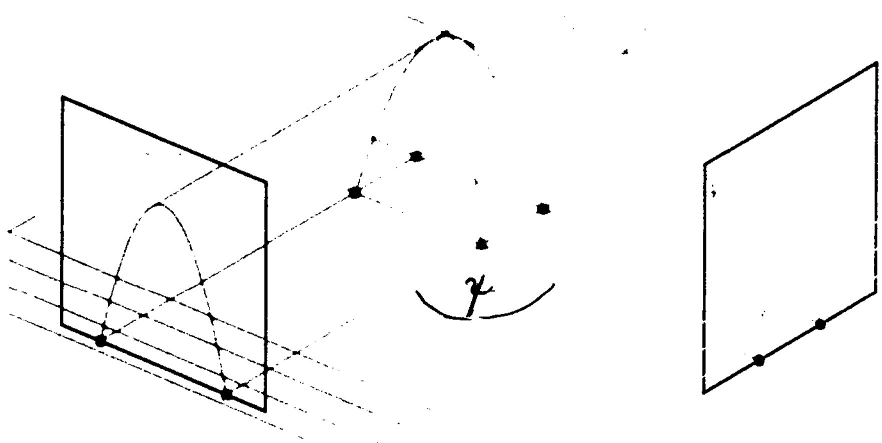
Emission distributed along line of sight → a large number of other views required to resolve F(d).

Stereoscopy of coronal loops

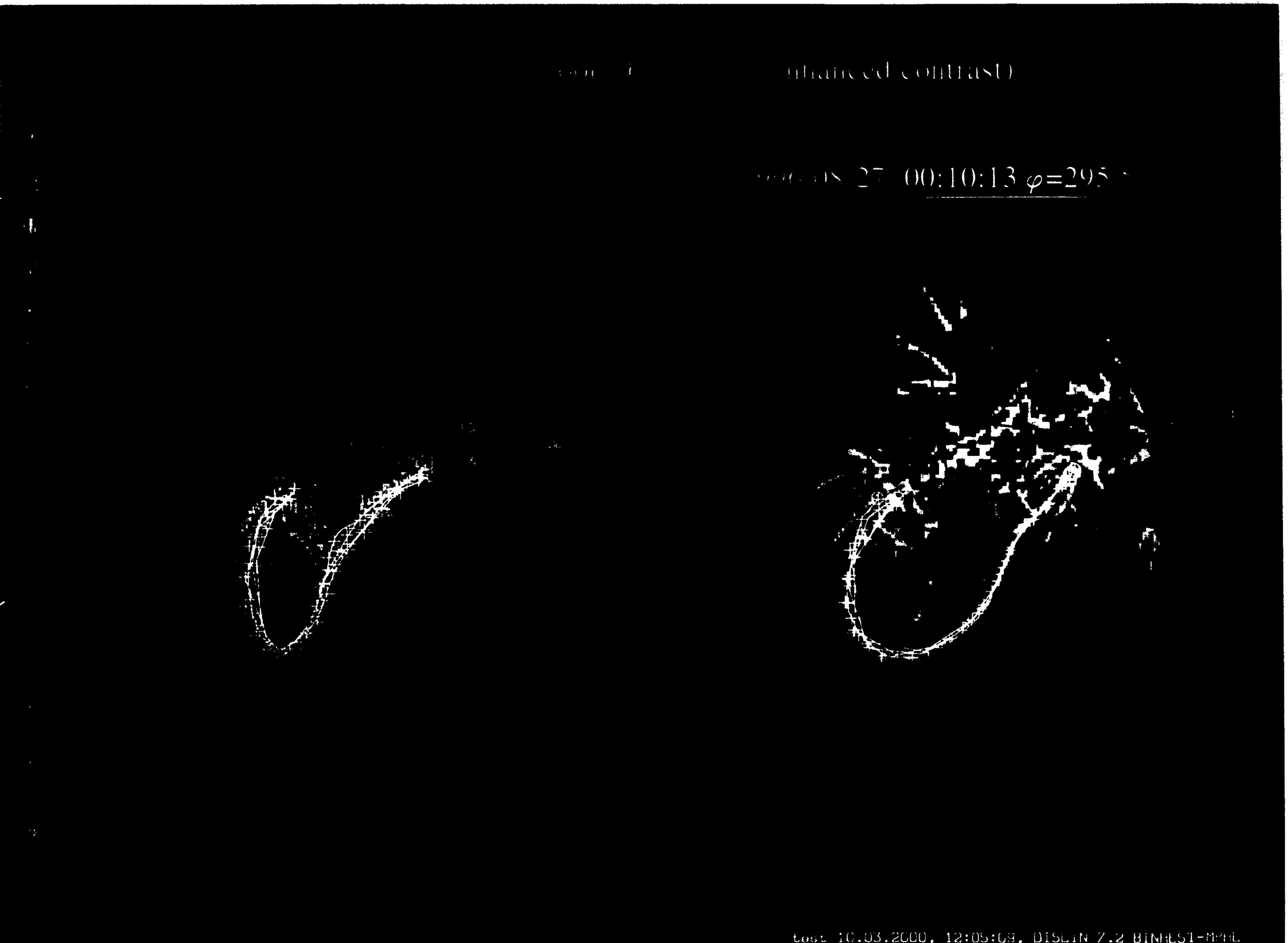
- **1D objects, e.g., thin coronal loops are best suited for a stereoscopic reconstruction**



- **Ambiguities arise whenever an epipolar plane is crossed more than once:**



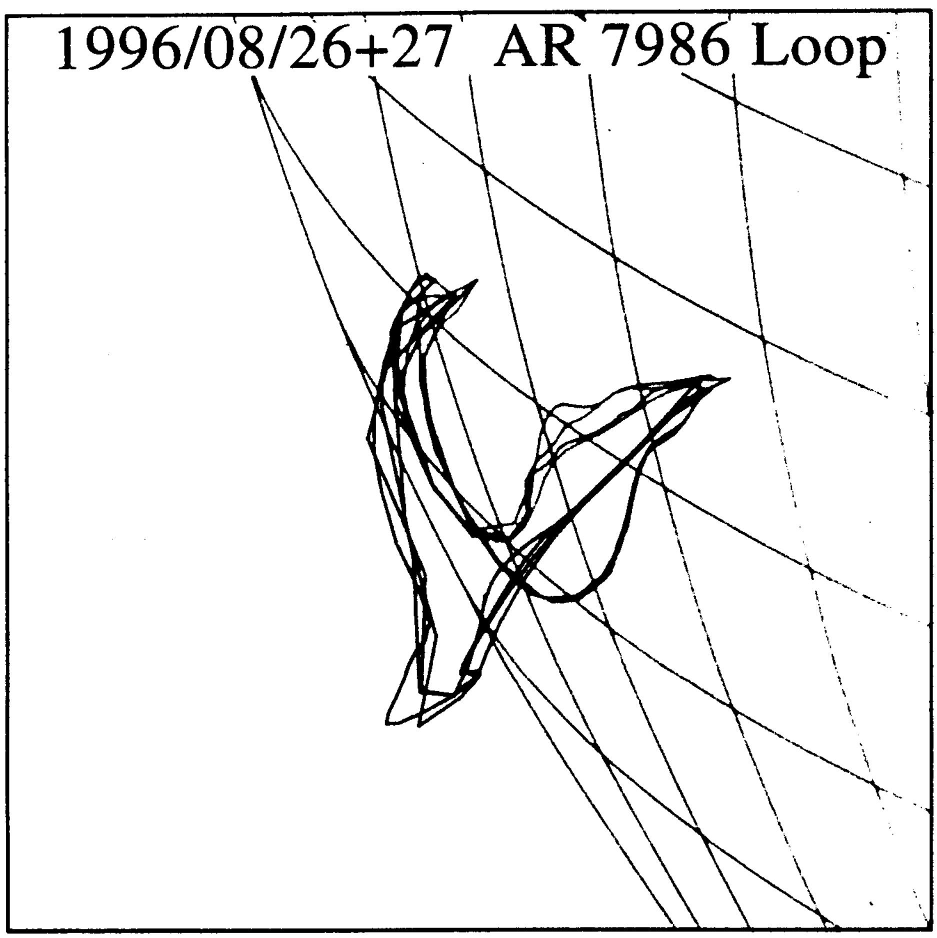
The geometrical error is $\simeq$ (pixel size)/sin($\phi/2$) where ϕ = angle between projection surface normals at intersection. The error is particularly large where ϕ is small and the projection surfaces are tangential to the epipolar plane.



contrast)
00:10:13 φ=295
test 10.03.2000, 12:05:08, DISLIN 7.2

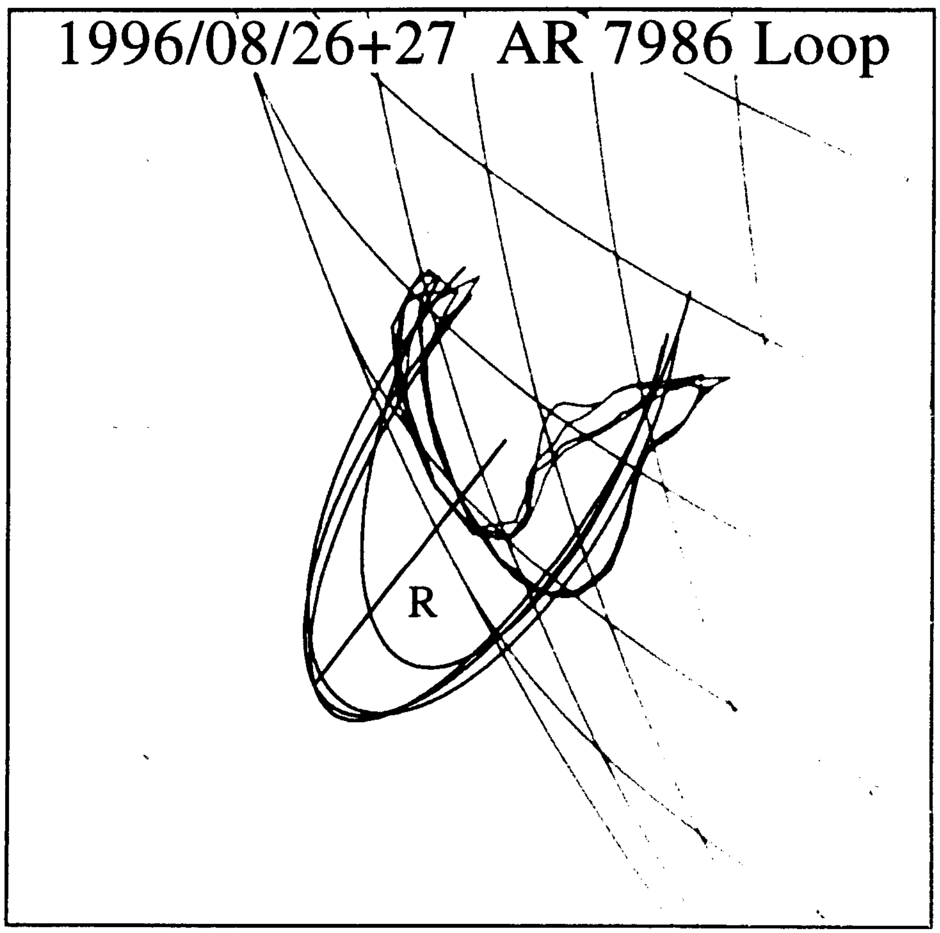
Stereo reconstruction I

- A rigorous stereoscopic reconstruction leads to "noisy" loop shapes:



Stereo reconstruction II

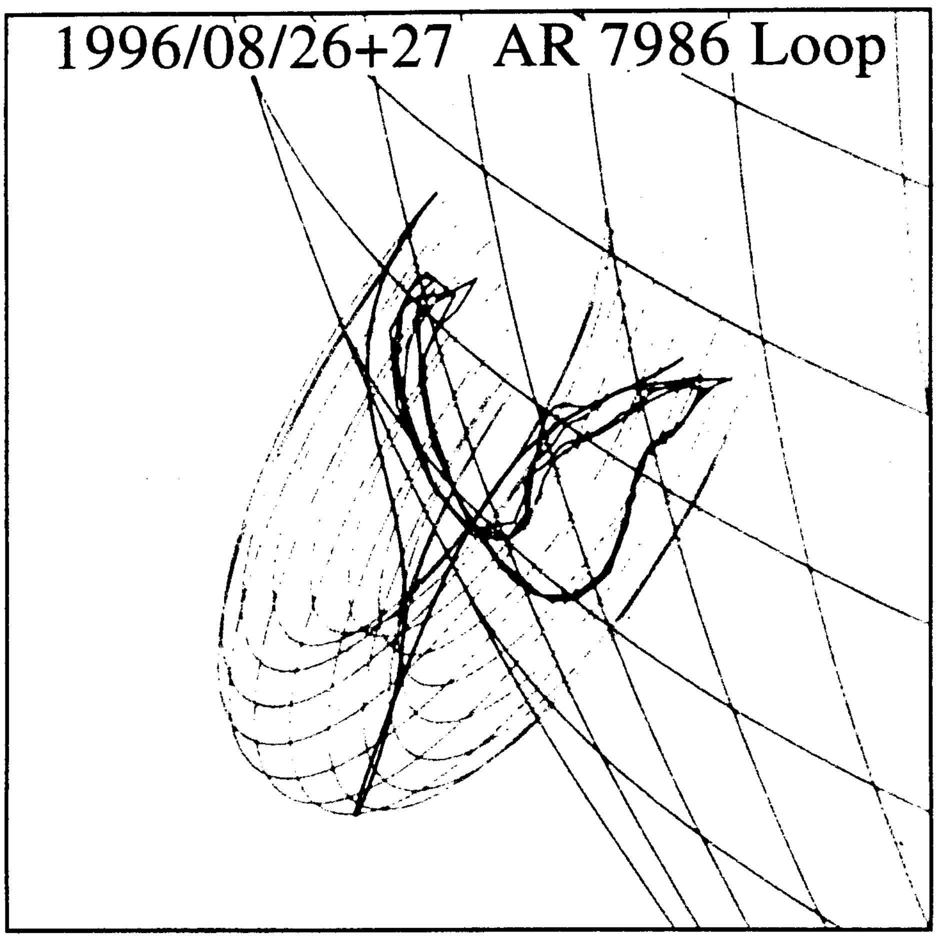
- Smoother results are obtained if the loop shape is constrained. The simplest constraint is a loop of circular shape. The plane of the circle, its radius and center have to be determined:



(Aschwanden et al., 1999)

Stereo reconstruction III

- As an extension of this fitting procedure further parameters may be introduced: the loop is constrained to the surface of a circular torus. Additional parameters: the small torus radius, the twist number and its phase:



(Portier-Fozzani et al., 1999)

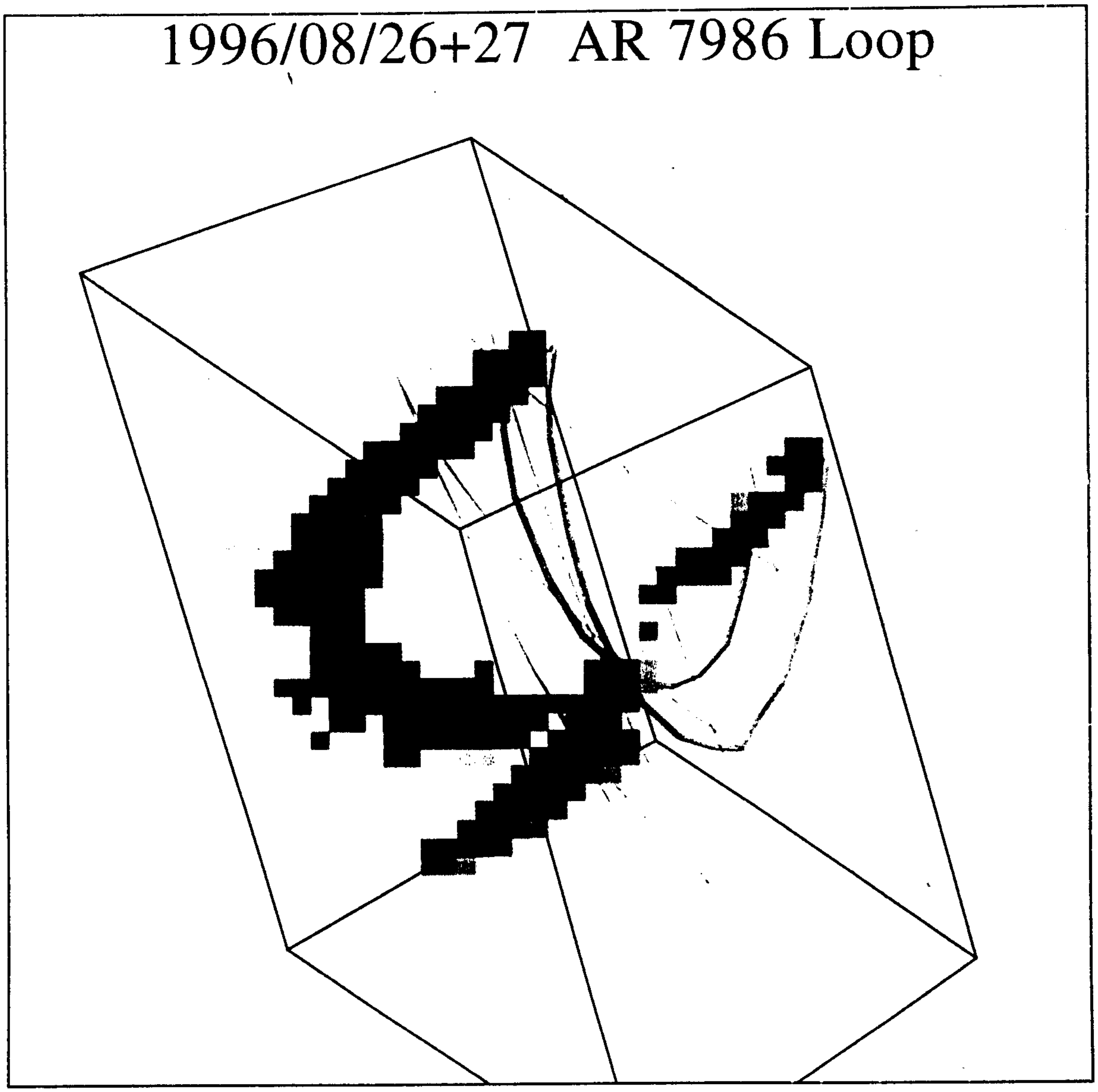
Simple sheared loops can be modelled with a twist number of 1/2. Experience has to show whether we need an even more general family of loop shapes.

Stereo reconstruction IV

- Modified backprojection: for two images a straightforward solution is:

$$m(x) = \frac{|\sin(\varphi_2 - \varphi_1)|}{\int p_i(s, \varphi_i)\ ds}\ p_1(x \cdot e^{\perp}_{\varphi_1}, \varphi_1) p_2(x \cdot e^{\perp}_{\varphi_2}, \varphi_2)$$

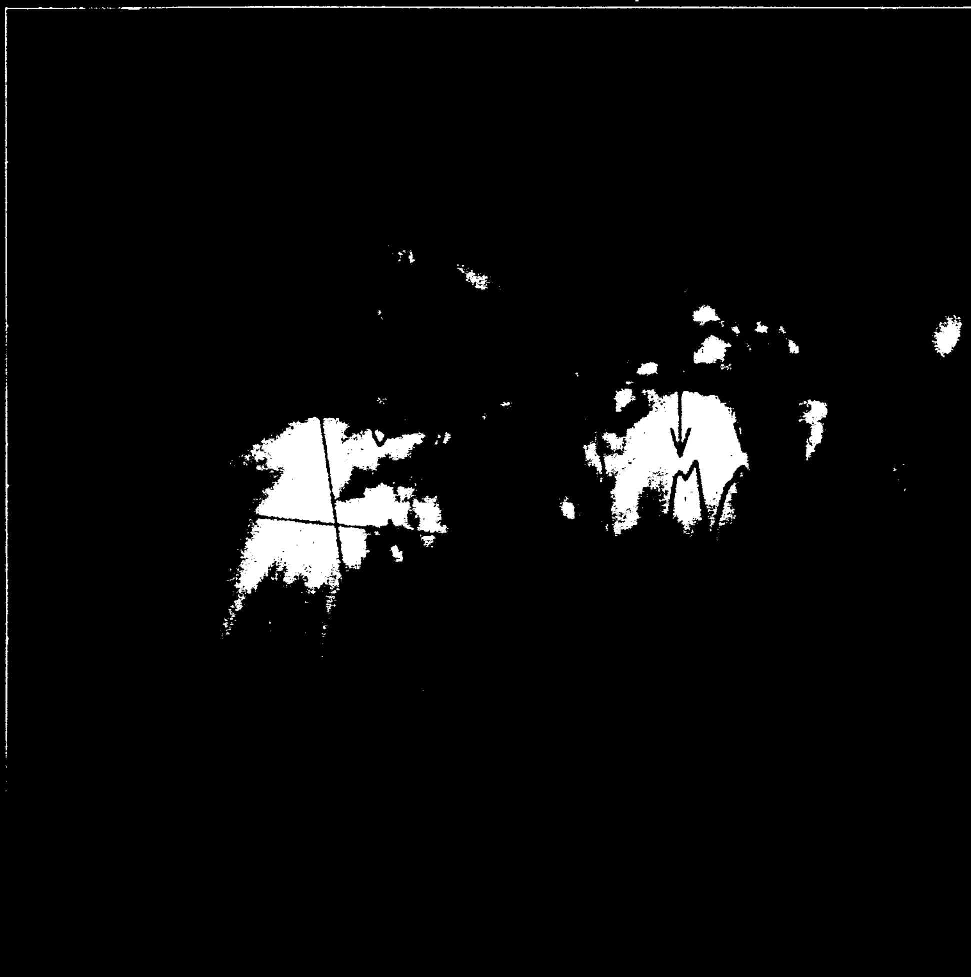
For an isolated loop we obtain:



- The solution contains all possible ambiguities. Therefore, a generalization for $>$ 2 images is required.

towards automatisation: correlation along the epipolar plane

1996/08/28 00:20:14 $\varphi=282.2$



1996/08/29 00:15:15 $\varphi=269.1$

Magnetic modelling

From solar surface magnetogram data we intend to model the coronal magnetic field up to heights where $\beta \ll 1$. The field will therefore be considered force free:

$$\boldsymbol{j} = \nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$$

The basic options will be:

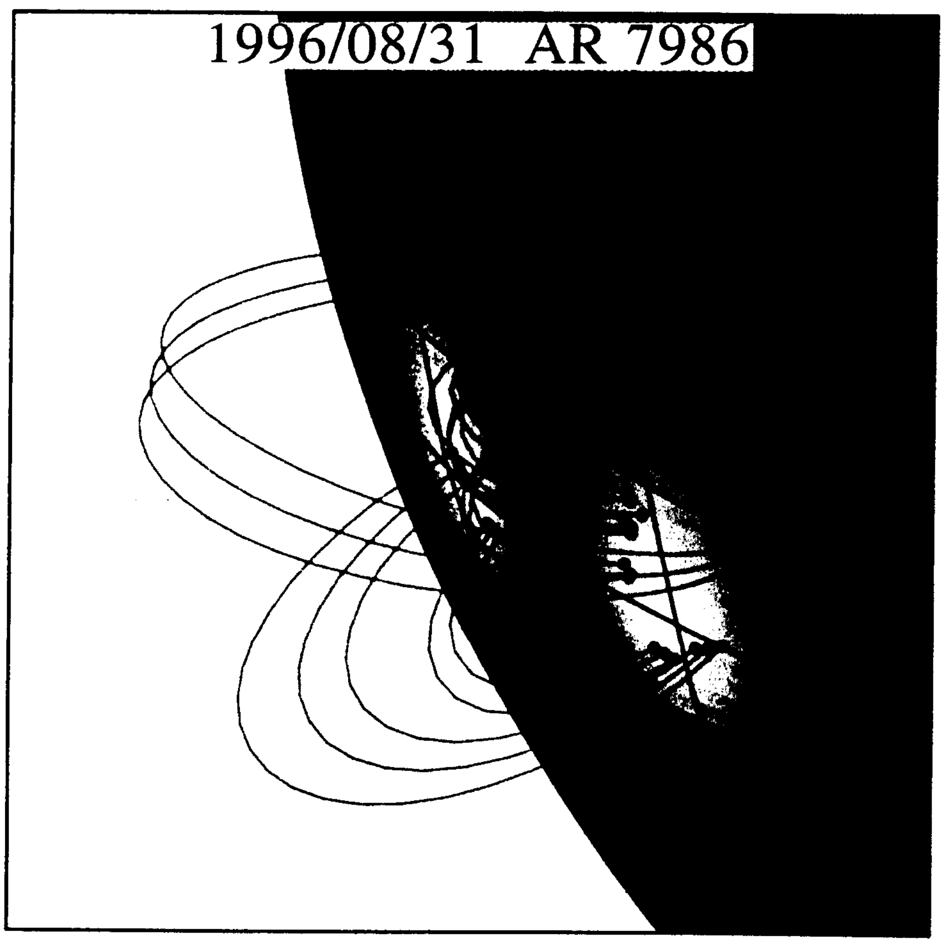
- $\alpha = 0$ gives the Laplace field model associated with the surface observations. Surface field is included by either spherical harmonics decomposition or Green's function integration.

- $\alpha \in \mathbb{R} - \{0\}$ gives a constant - α (or Taylor) field model. Surface field is included as above.

- $\alpha = \alpha(\boldsymbol{x})$ with $\boldsymbol{B} \cdot \nabla \alpha = 0$. Gives the most general force free field model. Will be solved iteratively in a bounded domain with the surface field as part of the boundary condition (Amari et al., 1999).

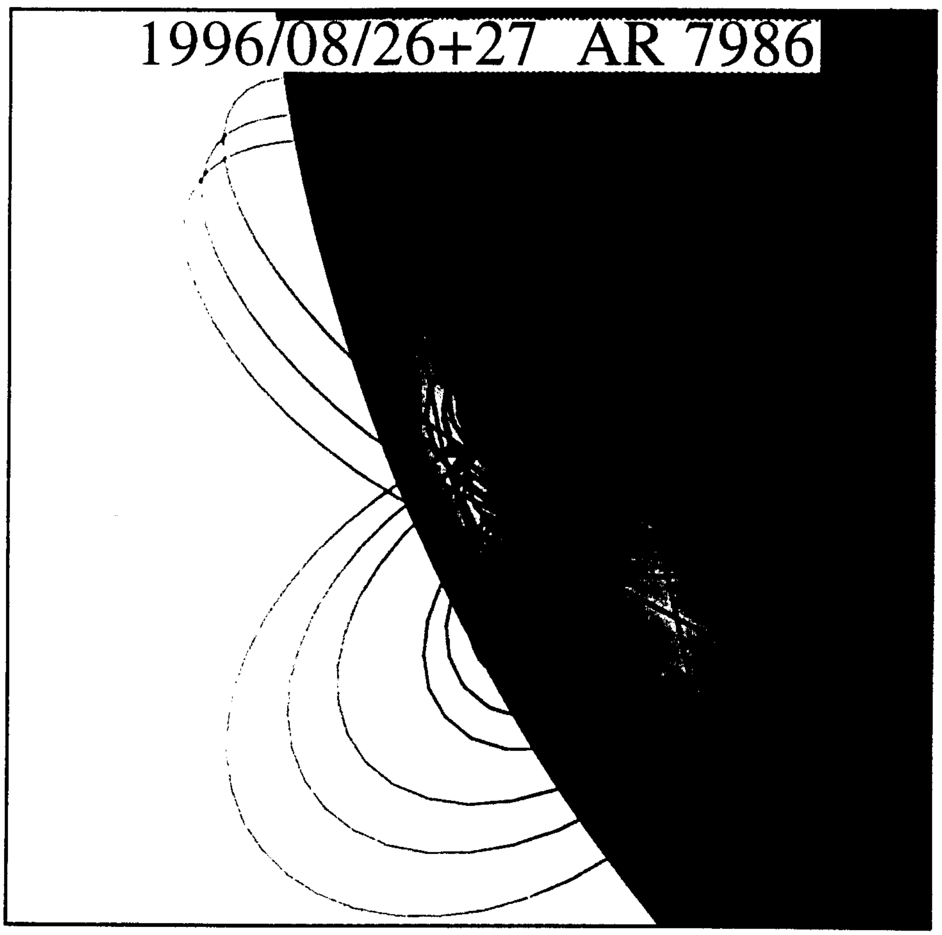
Magnetic modelling I

- A few selected field lines from an active region based on a Laplace field model:



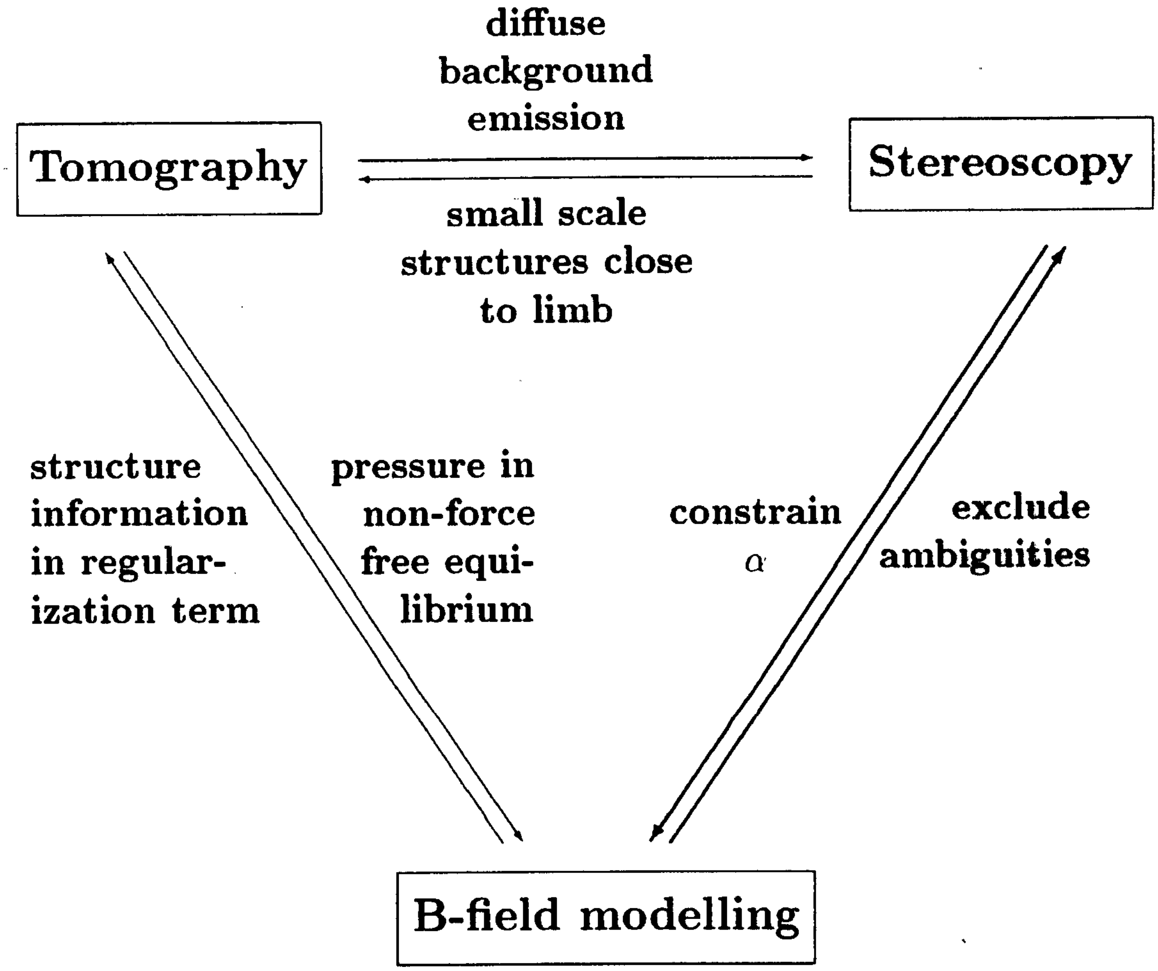
Magnetic modelling II

- **The same field lines from an active region for a force-free field model with constant $\alpha = 0.7\ \mathrm{R}_\odot^{-1}$:**



Summary

The three areas we have chosen to work on are mutually dependent:



We will try to enable the three packages to interchange their results so that the modelling efforts can be based on a more comprehensive data set and user response is minimized.